The watt balance: determination of the Planck constant and redefinition of the kilogram

Michael Stock, BIPM

Royal Society Discussion Meeting: The new SI

Outline

- **The present SI definition of the kilogram**
  - shortcomings of the present definition
  - possible alternative: link to fundamental constants
- **Watt balance experiments**
  - principle of operation
  - existing watt balances
- **Outlook to the redefinition of the kilogram**
  - present knowledge of the Planck constant *h*
  - status of the redefinition
  - future dissemination of the kilogram

The definition of the kilogram in the SI

The kilogram is the unit of mass;
it is equal to the mass of the international prototype of the kilogram.

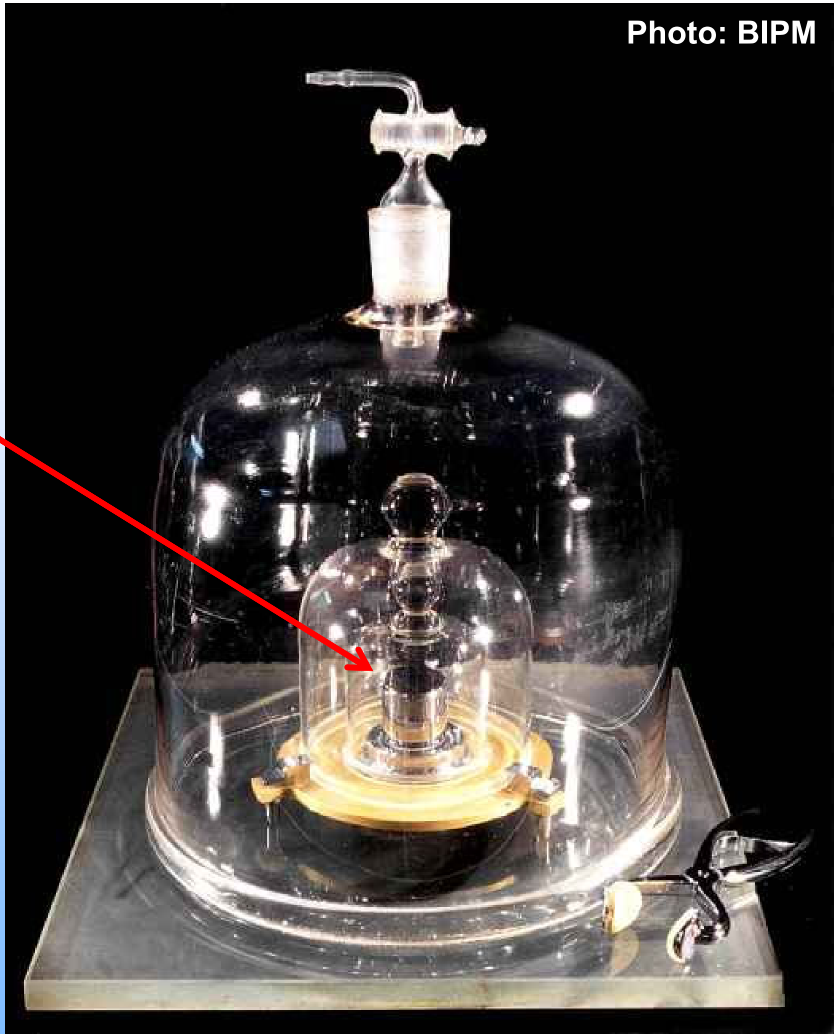
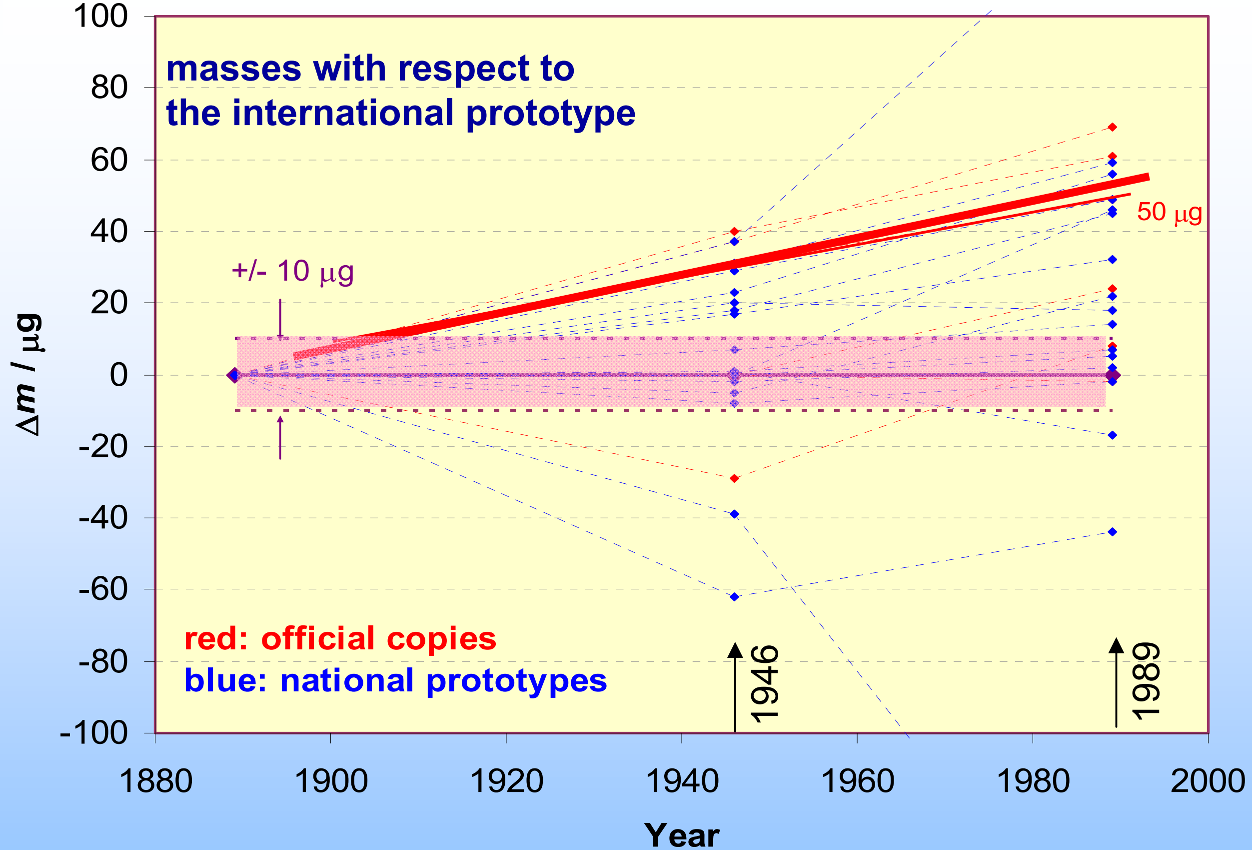


Photo: BIPM

- represents the mass of 1 dm^3 of H_2O at maximum density (4 °C)
- manufactured around 1880, ratified in 1889
- alloy of 90% Pt and 10% Ir
- cylindrical shape, Ø = *h* ~ 39 mm
- kept at the BIPM in ambient air

The kilogram is the last SI base unit defined by a material artefact.

Calibration history of the oldest national prototypes



Variations of about **50 µg (5×10^{-8}) in the mass of the standards over 100 years, that is 0.5 µg / year**

Masses of same material can be compared to within 1 µg

A drifing kg also influences the electrical units

Is the IPK losing mass or are the check standards getting heavier ? ?

► Redefinition of the kg in terms of a fundamental constant of nature, for example Planck constant *h* (advantageous for electrical metrology)

Example of a possible new definition of the kg

"The kilogram, unit of mass, is such that the Planck constant h is exactly equal to 6.626 068 96 $\times 10^{-34}$ joule second:

$$h = 6.626\,068\,96 \times 10^{-34}\ \mathrm{J\,s}$$"

$$h = 6.626\,068\,96 \times 10^{-34}\ \mathrm{kg\ m^2\ s^{-1}}$$

The value of h is fixed by nature

The numerical value of h is fixed by the definition of the kg

The effect of this equation is to define 1 kg

The units m and s are defined in the SI

The numerical value needs to be determined in the present SI, to avoid significant discontinuities.

Why do we need watt balance experiments ?

A watt balance allows to establish a link between *h* and a macroscopic mass.

Watt balances are needed for several objectives:

- **Determination of *h* with uncertainty of the order of 1 part in 10^8 in the present SI, several independant results desirable;**
- **Realization of the new definition of the kg after the redefinition (long-term task). Several instruments needed;**
- **Long-term study of the drift of the international prototype.**

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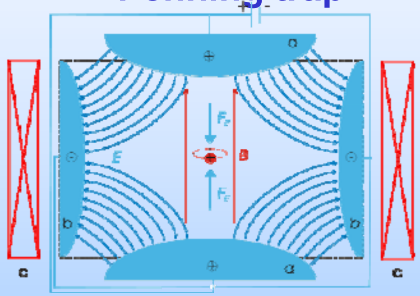
The basic problem: linking the macroscopic and the microscopic world

► macroscopic masses at the level of 1 kg can be compared with an uncertainty of about 1 part in 10^9 (1 μg).

mass spectrometers, Penning trap



► atomic masses can be compared with an uncertainty typically less than 1 part in 10^9 in a range of [0.00055 *u*, 100 *u*].

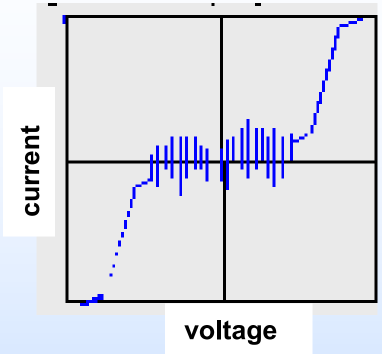
► but: how to compare a macroscopic mass with a microscopic mass (m_e) or a fundamental constant (h)?

Solution:	- macroscopic electrical quantum effects - equivalence of electrical and mechanical power

Macroscopic electrical quantum effects

Josephson effect (1962)

(B. Josephson, Nobel Prize 1973)



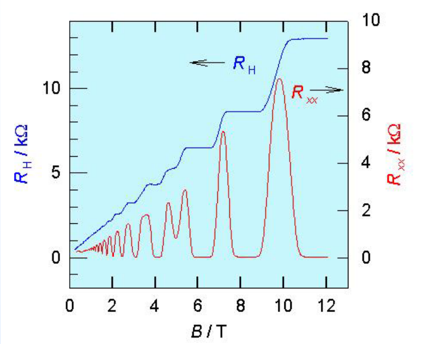
$$U_J(n) = \frac{n\,f}{K_J}, \quad K_J = \frac{2e}{h}$$

unc. of K_J: 2.5 x 10^{-8} (2006)

reproducibility at level of 10 V: < 10^{-10}

Quantum-Hall effect (1980)

(K. von Klitzing, Nobel Prize 1985)



$$R_H(i) = \frac{R_K}{i}, \quad R_K = \frac{h}{e^2}$$

unc. of R_K : 7 x 10^{-10} (2006)

reproducibility at level of 100 Ω: approx. 10^{-9}

Both effects link macroscopic measurands (voltage, resistance) with fundamental constants (*h* and *e*).

Derivation of the watt balance equation

- Electrical power can be expressed as

$$P_{\mathrm{el}} = U\, I = \frac{U_1\, U_2}{R}$$

Josephson effect $U_{\mathrm{J}}(n) = n\, f \left(\frac{h}{2\, e} \right)$

quantum Hall effect $R_{\mathrm{H}}(i) = \frac{1}{i} \left(\frac{h}{e^2} \right)$

$P_{\mathrm{el}} = C_{\mathrm{el}}\, f_1\, f_2\, h$ **electrical power now depends on *h***

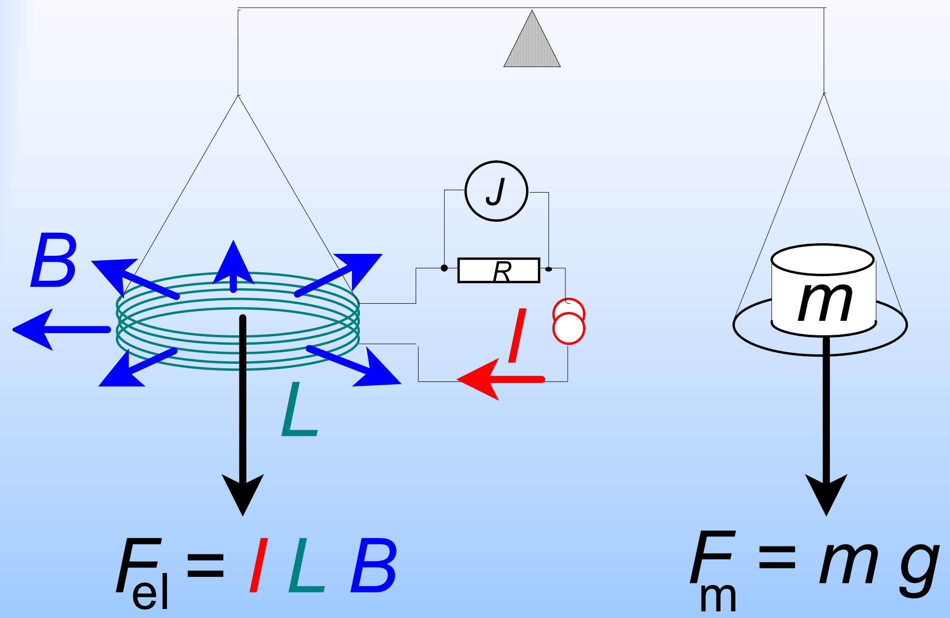
- Electrical and mechanical power are equivalent
  - are quantities of the same type
  - are measured with the same unit

$P_{\mathrm{m}}(m, v, g, \ldots) = P_{\mathrm{el}} = C_{\mathrm{el}}\, f_1\, f_2\, h$ **mass *m* and *h* appear in the same equation**

- Avoid direct energy/power conversion !

Watt balance principle - 1

Phase 1: static experiment



Weight of a test mass is compared with the force on a coil in a magnetic field.

$$mg = -I\frac{d\Phi}{dz}$$

In a radial magnetic field, this can be simplified to

$$m\ g = I\ L\ B$$

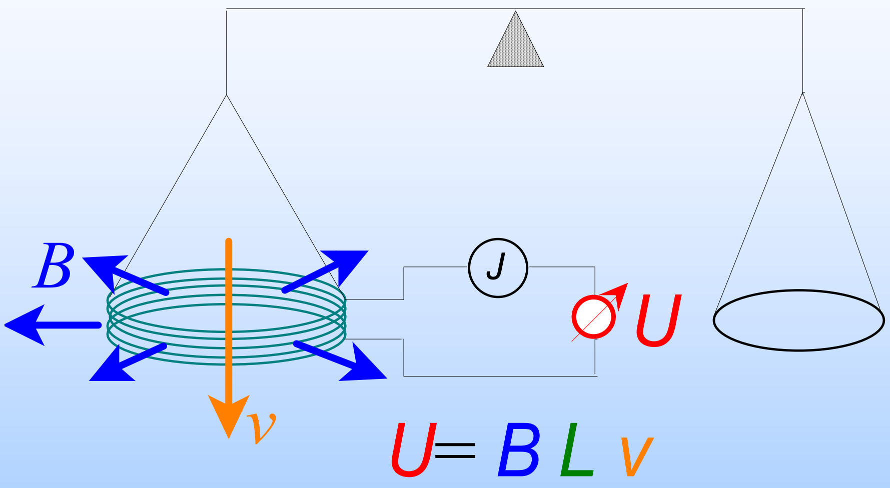
current

flux density

wire length

Watt balance principle - 2

Phase 2: dynamic experiment



Coil is moved through the magnetic field and a voltage is induced.

$$U = -v\frac{d\Phi}{dz}$$

In a radial magnetic field, this can be simplified to

$$U = B\,L\,v$$

- U: ind. voltage
- B: flux density
- L: wire length
- v: velocity

Watt balance equations

static phase: $m\,g = I\,B\,L$

dynamic phase: $U = v\,B\,L$

If *L*, *B* constant: $U\,I = m\,g\,v$ $\quad \cancel{B\,L}$

$$P_{\mathrm{el}} = P_{\mathrm{mech}}$$

Watt balance does not realize a <u>direct conversion</u> of electrical and mechanical energy

Energy losses due to dissipative processes (friction,…) do not enter into the measurement equation.

Link between the kg and the Planck constant

U and *R* are measured using Josephson effect and the quantum Hall effect

$$U\ I = \frac{U_1\ U_2}{R} = C_{\mathrm{el}}\ f_1\ f_2\ h$$

⇩

$$U\ I = m\ g\ v$$

⇨

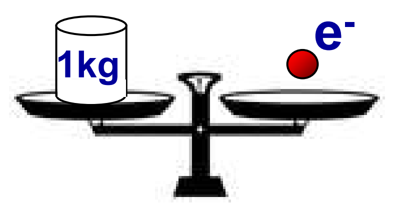
$$h = \frac{m\ g\ v}{C_{\mathrm{el}}\ f_1\ f_2}$$

A new definition of the kg requires the measurement of *h* with an uncertainty of some parts in 10^8.

Another interpretation: weighing the electron

Watt balance equation:

$$h = \frac{m\,g\,v}{C_{\mathrm{el}}\,f_1\,f_2}$$



Definition of the Rydberg constant:
(theory of hydrogen spectrum)

$$R_\infty = \alpha^2 \frac{m_{\mathrm{e}}\,c}{2\,h} \quad \longrightarrow \quad m_{\mathrm{e}} = \frac{2\,h\,R_\infty}{\alpha^2\,c}$$

($u_r(R)$= 7 x 10^{-12}, $u_r(\alpha)$=7 x 10^{-10})

Most accurate determination of the electron mass to date !

($u_r(m_e) = u_r(h)$ = 5 x 10^{-8}), m_e = 9.109 382 15 (45) x 10^{-31} kg

Existing watt balance experiments

2009

NPL, 1976,	“first watt balance”
NIST, 1980,	“biggest watt balance“, with superconducting magnet
METAS, 1997,	“smallest watt balance“
LNE, 2001,	“moving beam watt balance“
BIPM, 2003,	“single mode watt balance“, plans for superconducting watt bal.
NIM, 2006,	“mutual inductance joule balance“
NRC, 2009	

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The NPL watt balance – Mark II

Watt balance principle was proposed by B. Kibble, NPL in 1976

Work on Mark II started around 1990

balance beam (1.2 m) on knife edge

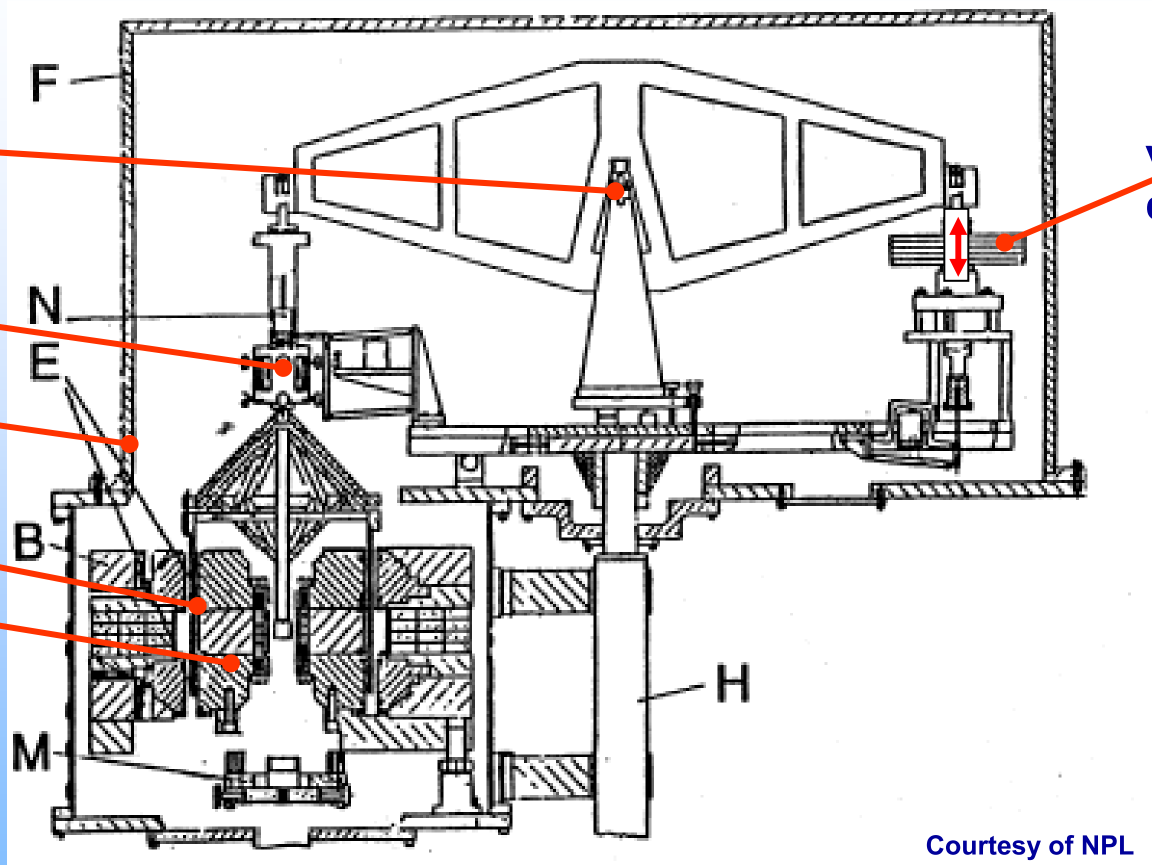
test mass
1 kg, 0.5 kg

vacuum chamber

coil

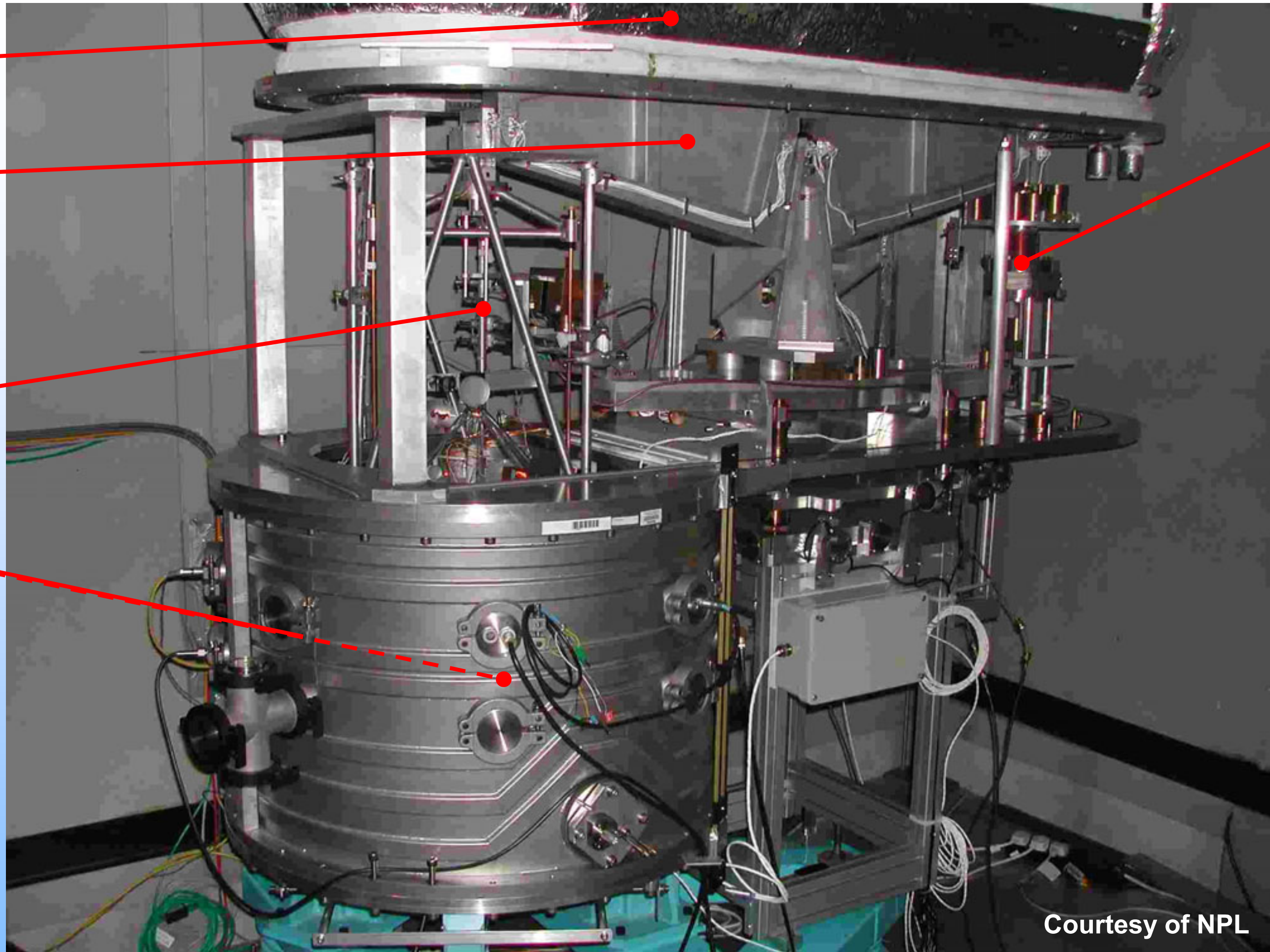
permanent magnet
radial field 0.42 T

velocity drive coil

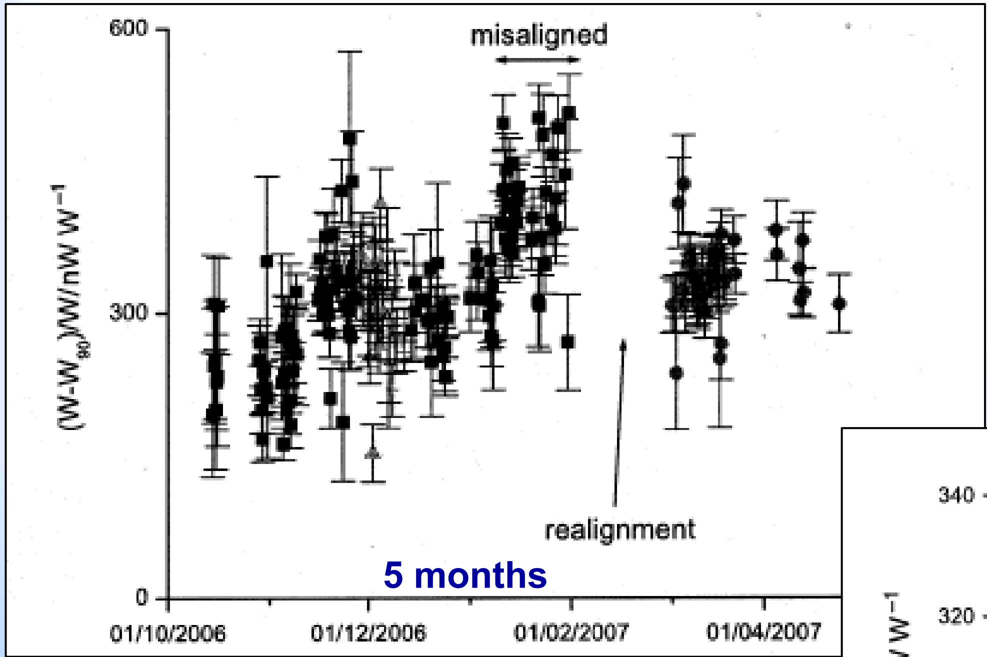


Courtesy of NPL

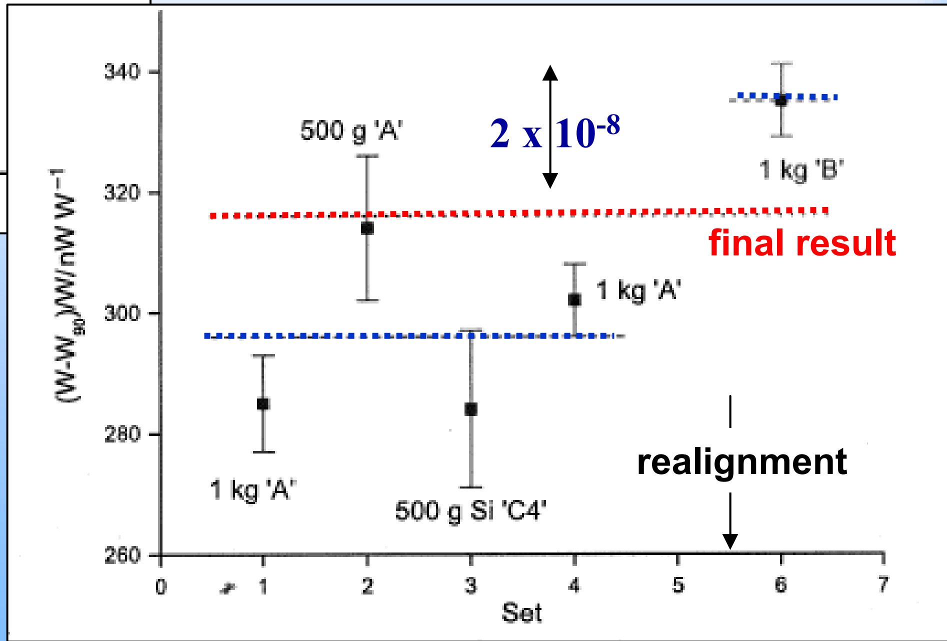
The NPL watt balance – Mark II



The NPL watt balance – Mark II, 2007 result



data treated in six groups



h = 6.626 070 95 (44) x 10^{-34} Js

u_r (h, 2007) = 7 x 10^{-8}

I.A. Robinson, B.P. Kibble, *Metrologia*, 2007, 44, n° 6, 427-440.

u_r (h, 2010) = 20 x 10^{-8}

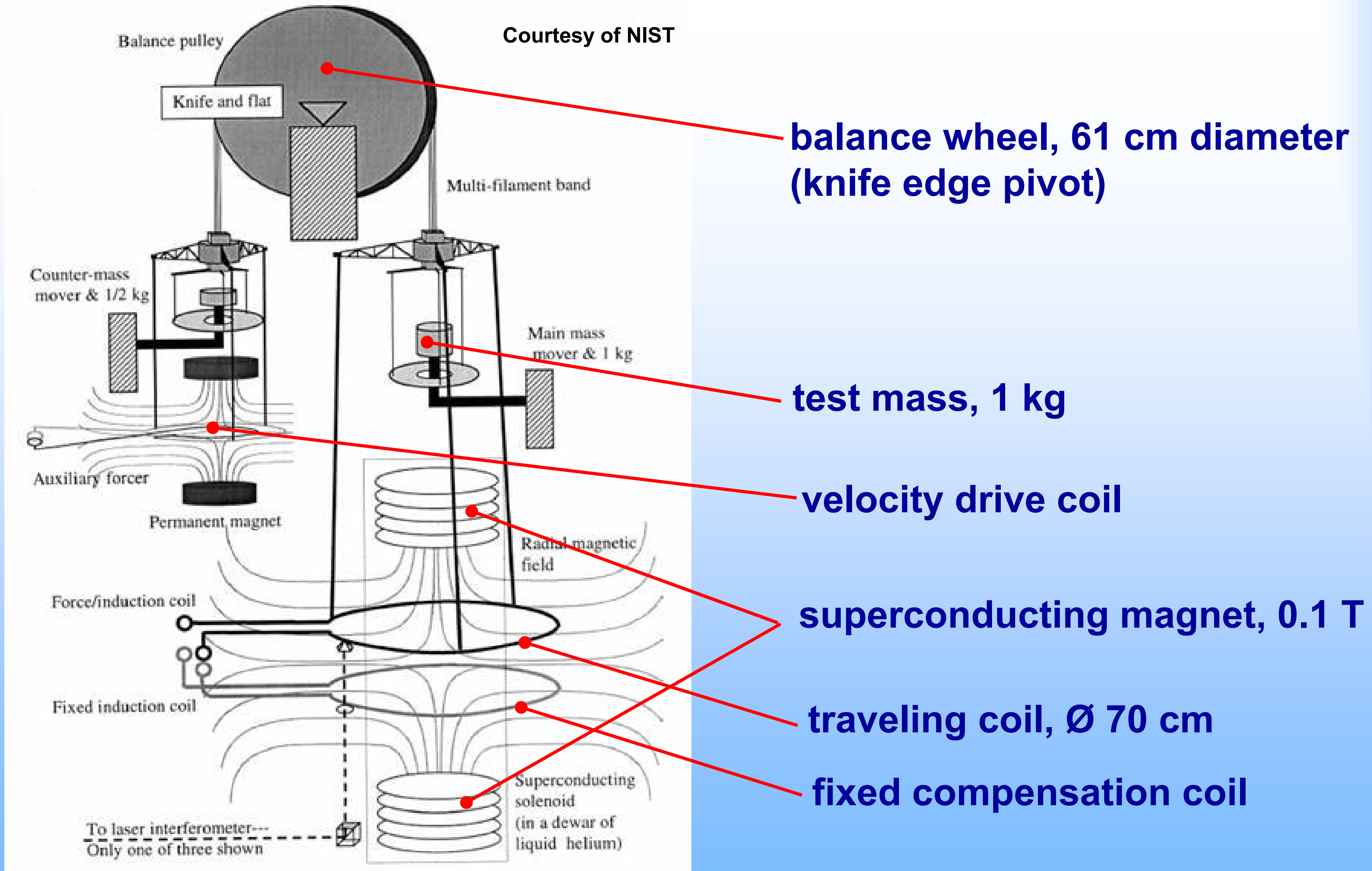
NPL watt balance, starting a new life as NRC watt balance

Shipped in summer 2009 from NPL, Teddington, to NRC, Ottawa

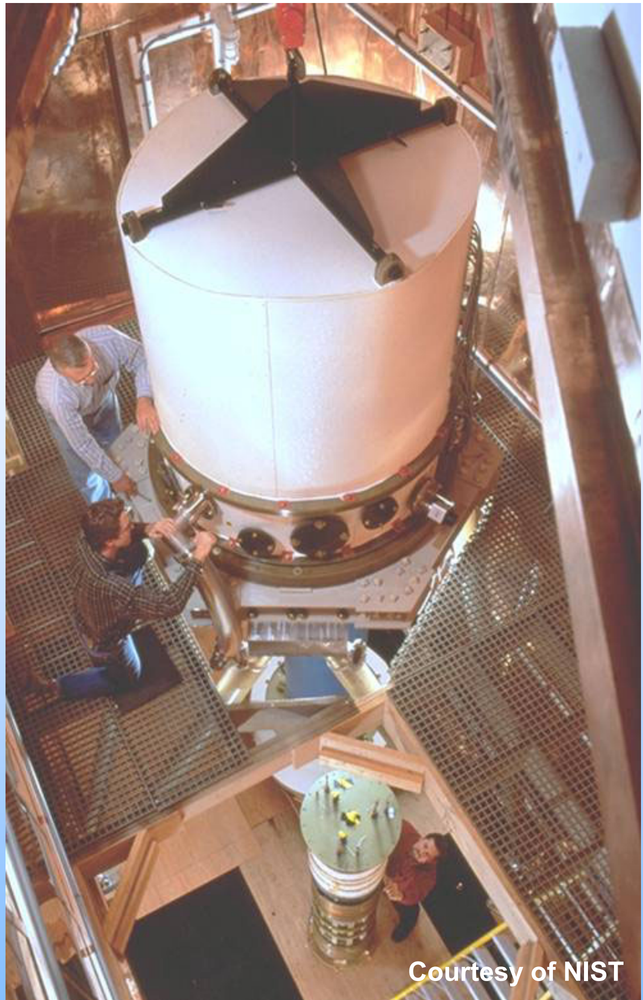
Operational at the several ppm level
Several parts being rebuild

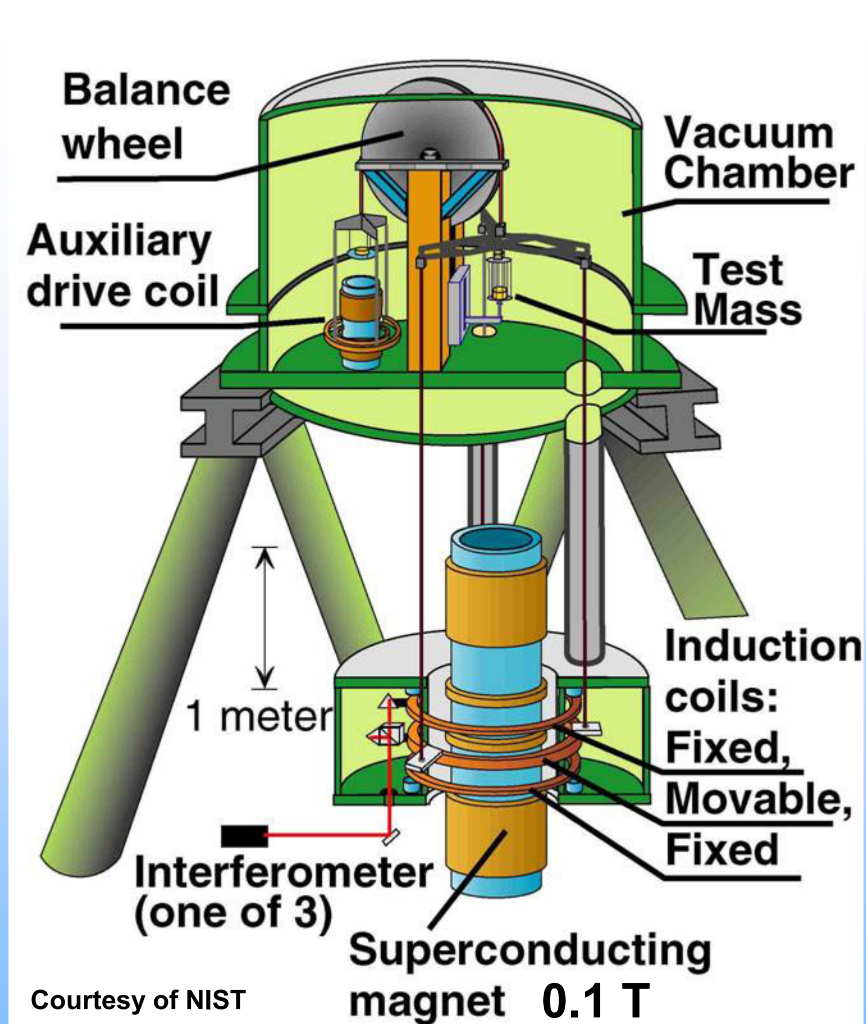
$u_r < 10^{-7}$ expected for mid 2011

The NIST watt balance – started 1980



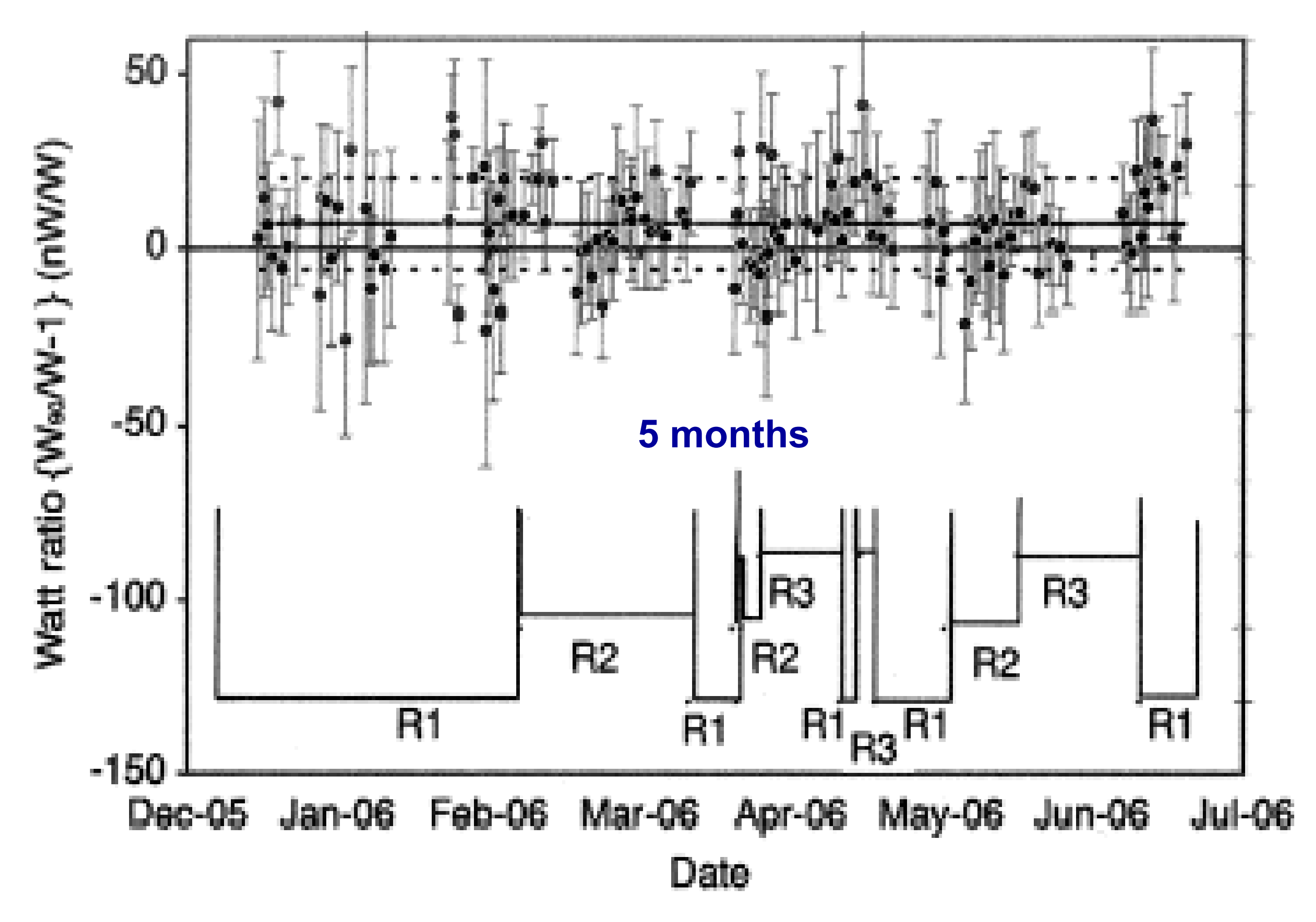
The NIST watt balance





u_r (h, 1998) = 9 x 10^{-8} u_r (h, 2005) = 5 x 10^{-8} u_r (h, 2007) = 3.6 x 10^{-8}

The NIST 2007 result – the lowest published uncertainty



u_r (h, 2007) = 3.6×10^{-8}

R. Steiner, E. Williams et al., *IEEE Trans. Instrum. Meas.* 56, n° 2, 2007

NIST uncertainty budget

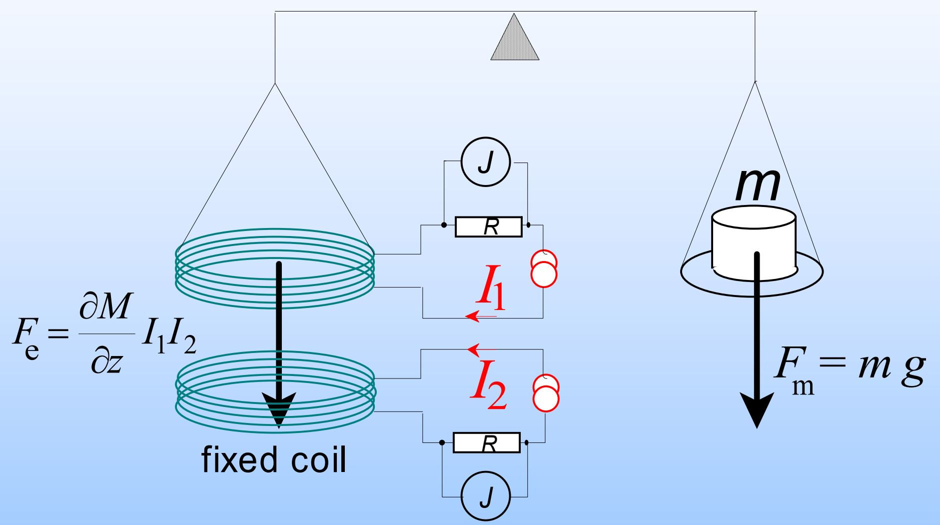
Improvements to the most significant type B uncertainty contributions

Uncertainty contribution (nW/W)	2005	2006	
Mass	15	10	
Resistance	10	10	
Local gravity acceleration	30	12	Improved gravity transfer
Wheel surface flatness	20	2	New support band, and improved investigation of effect
Electrical grounding	12	15	
Laser wave front shear	10	10	
Mass std. magnetic susceptibility	11	7	
Fitting order, plc change	16	16	
16 others	13.1	12.3	
RSS combined	**46.8**	**33.2**	

The NIM watt balance – started in 2006

New approach: no dynamic phase needed

Based on mutual inductance between two coils



$$mg = \frac{\partial M}{\partial z} I_1\ I_2$$

geometric factor: To be determined by separate experiment

The fixed coil carries a current I_2 to produce a magnetic field.

The second coil with current I_1 hangs on an arm of the balance.

The NIM watt balance – started in 2006

$$mg = \frac{\partial M}{\partial z} I_1 \, I_2$$

Integration leads to:

$$mg\left(z_2 - z_1\right) + \left[M\left(z_1\right) - M\left(z_2\right)\right] I_1 \, I_2 = \int_1^2 \Delta f_z\left(z\right) dz$$

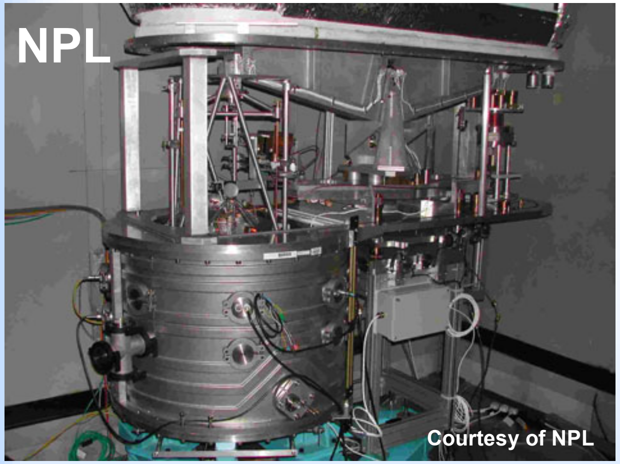
change of potential energy

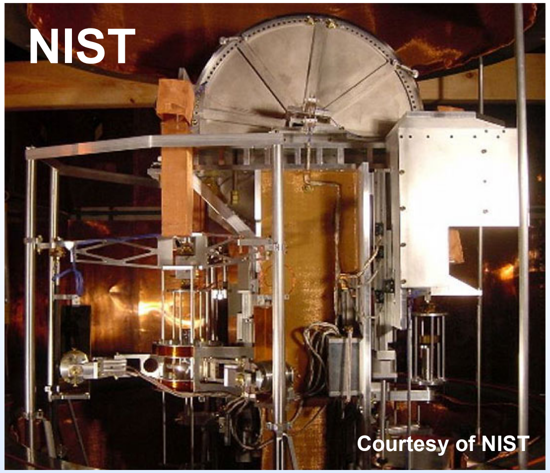
change of magnetic energy

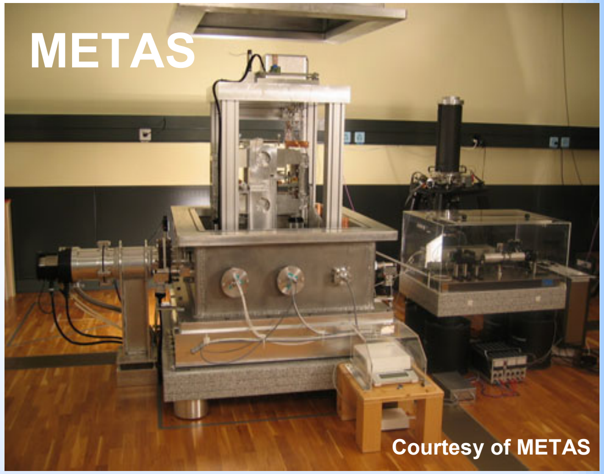
Present state

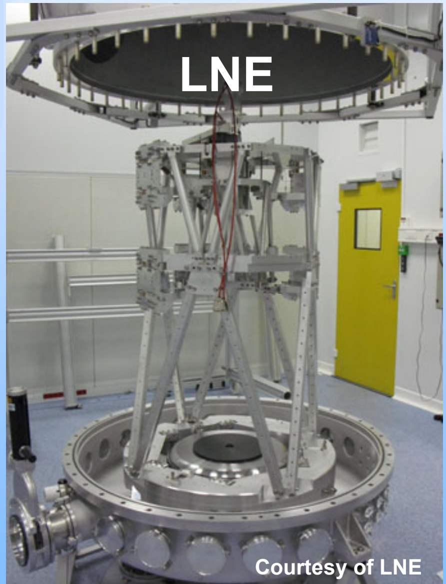
- work was initially focused on measuring *M* using an innovative approach based on direct digital synthesis
- mutual inductance has been measured to 1 part in 10^7, but difficult to improve
- the magnetic field is very small, they plan to use superconducting coil
- balance has been purchased in mid-2010

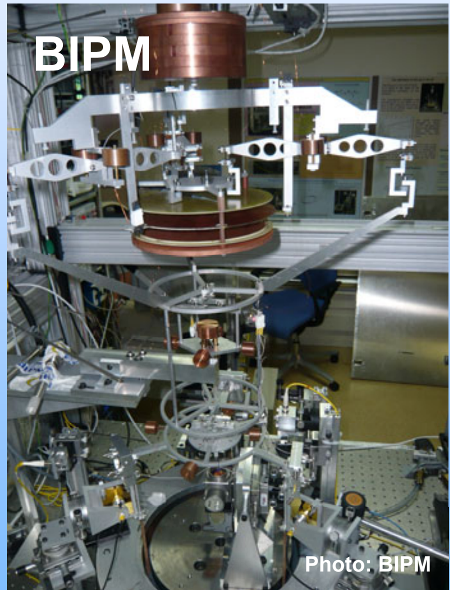
Photo gallery of all watt balances

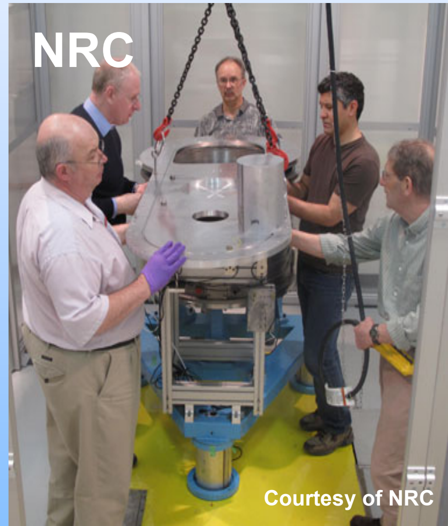












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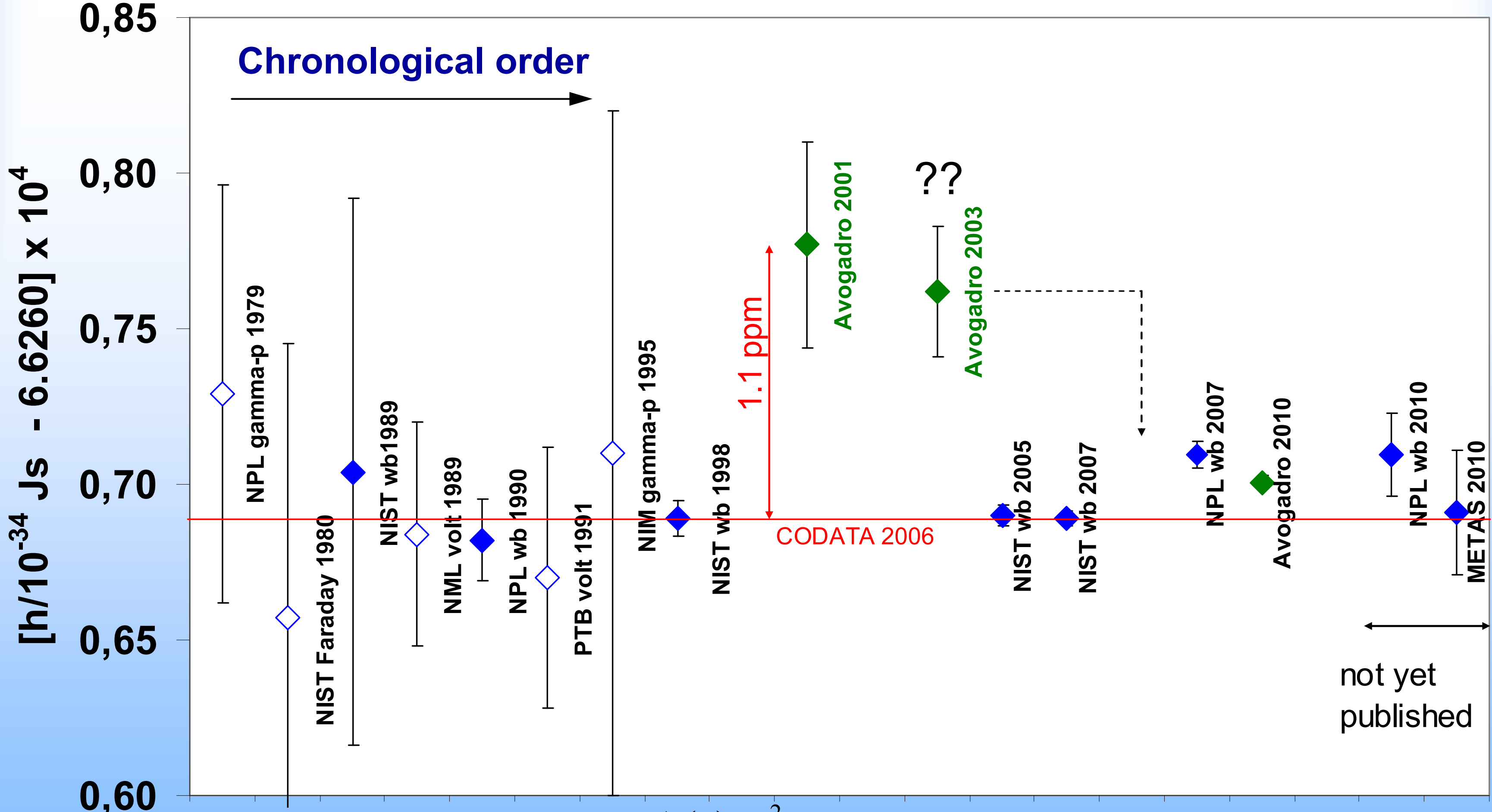
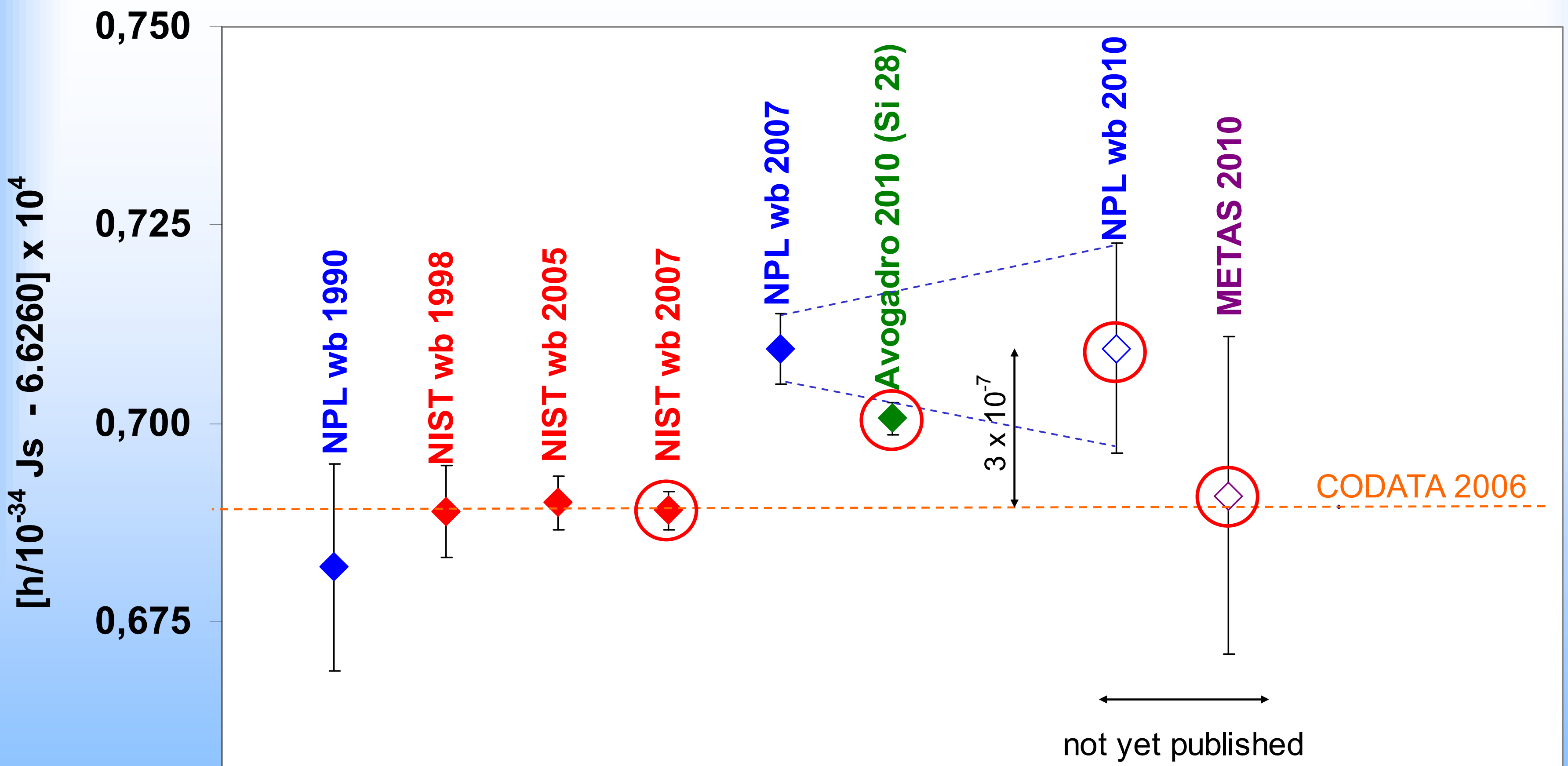
Available data for Planck constant



N N_A **and *h* are related by** $N_A\, h = \dfrac{c}{2}\dfrac{A_r(e)\,\alpha^2}{R_\infty} M_u$ $\qquad u_r(N_A\, h) = 2\times u_r(\alpha) = 1.4\times10^{-9}$

Available data for Planck constant (only WB and Avogadro results)



Requirements for a redefinition of the kilogram

The Consultative Committee for Mass (CCM) recommends that the following conditions be met:

- **At least three independent experiments, including watt balances and int. Avogadro Coordination project with $u_r \leq 5 \times 10^{-8}$**

- **At least one of these shall have $u_r \leq 2 \times 10^{-8}$**

- **All results shall agree within 95 % level of confidence**

- **A sufficient number of facilities for robust realization of the new definition are needed after the redefinition**

⇨ **Conditions are not fulfilled in 2011, no redefinition at 2011 CGPM**

Next occasion will be CGPM in 2015

Future developments

Internat. Avo. Collab. (^{28}Si-sphere),	u_r close to 2 x 10^{-8} planned for 2011-2012
NIST watt balance	unc. not likely to improve a lot, new instrument being planned
NPL watt balance	final publication in preparation
METAS watt balance	new instrument being developped
LNE watt balance	first measurements end 2011, objective u_r close to 2 x 10^{-8} in 2014
BIPM watt balance	first meas. made, $u_r < 10^{-7}$ planned for 2015
NIM joule balance	under development
NRC watt balance	$u_r < 10^{-7}$ expected for mid-2011

Possible future dissemination of the kilogram

watt balance 1

watt balance 2

…..

watt balance N

N =3 …5 (?)

Other techniques (XRCD,…)

comparison: degree of equivalence

pool of artefacts

BIPM watt balance

dissemination of the kg

NMI 1

NMI 2

NMI 3

…..

long-term stability: watt balances

short-term stability: artefacts

More details in Richard Davis' talk this afternoon